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ANALYTICAL STRENGTH FORMULAS

FOR SHIP HULLS

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The subject of this report is a proposed new surface ship hull concept consisting of a double skin that wraps around the bottom, sides and main deck. The two skins are connected by plates normal to the surfaces, forming a cellular structure similar to a cardboard box. Modeling the ship hull as a circular cylindrical orthotropic shell surrounding an elastic core, we are able to obtain analytical formulas for estimating the principal stresses in such a structure, subjected to end bending moments and lateral pressure. These formulas indicate that the stiffness of the proposed bulkheads in the proposed design could be reduced by a factor of 10 without incurring significant secondary stresses in the double hull.

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ANALYTICAL STRENGTH FORMULAS

FOR SHIP HULLS

BY

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1. INTRODUCTION: NEW SHIP STRUCTURE AND MODEL

The proposed new surface ship hull concept consists of a double skin that wraps around the bottom, sides, and main deck. The two skins are connected by plates normal to the surfaces forming a cellular structure similar to a cardboard box, as shown in Figure 1. The advantages of this type of construction have been described by Okamoto, et. al. [1], and Beach [2].

The object of this report is to develop formulas for estimating the principal stresses in such a structure. In order to be able to obtain simple analytical formulas, we model a ship hull as a circular cylindrical orthotropic shell surrounding an elastic core. Under a pure end bending moment M and a lateral pressure loading q which does not vary along the length, the cylinder bends into a curved tube of oval cross section, as shown in Figure 2.

2. SHELL EQUATIONS AND SOLUTION

The theory governing the deformation of thin shells is well developed and available in many forms. Here we use the semi-momentless shell theory of Axelrad [3]. ** When specialized to the linear St. Venant problem for circular cylindrical shells, Axelrad's equations (2.122) - (2.123) reduce to

$$\frac{\partial^4 \kappa_2}{\partial \eta^4} + \frac{\partial^2 \kappa_2}{\partial \eta^2} = -\frac{R^2}{D_2} \frac{\partial^2 q}{\partial \eta^2} \tag{1}$$

$$\frac{\partial^4 T_1}{\partial \eta^4} + \frac{\partial^2 T_1}{\partial \eta^2} = 0 \tag{2}$$

Here the coordinate η denotes the circumferential angle and R denotes the radius of the undeformed cylinder, as shown in Figure 2. The principal values of the change in curvature tensor are denoted by $\kappa_1(\eta)$ and $\kappa_2(\eta)$; their geometrical definitions are illustrated in Figure 3. The longitudinal membrane force $T_1(\eta)$ (force per unit length of the shell midsurface) is related to the longitudinal strain $e_1(\eta)$ by the constituitive law

$$T_1 = Ehe_1 \tag{3}$$

Here h is the thickness of the shell, and E is Young's modulus. The longitudinal bending moment $M_1(\eta)$ and transverse bending moment $M_2(\eta)$ (moments per unit length of the shell midsurface) are related to the curvature changes $\kappa_1(\eta)$ and $\kappa_2(\eta)$ by the constituitive laws

$$M_1 = D_1(\kappa_1 + \nu\kappa_2), \quad M_2 = D_2(\kappa_2 + \nu\kappa_1) \tag{4}$$

** Our equations may also be obtained from other shell theories, such as that of Simmonds [4].

Here D_1 and D_2 are bending stiffnesses, and ν is Poisson's ratio. The longitudinal curvature change $\kappa_1(\eta)$ and transverse membrane force $T_2(\eta)$ are given in terms of the above variables by

$$\kappa_1 = -\frac{1}{EhR}\frac{\partial^2 T_1}{\partial \eta^2}, \quad T_2 = Rq + \frac{1}{R}\frac{\partial^2 M_2}{\partial \eta^2} \tag{5}$$

The net bending moment M acting on any cross section of the tube may be calculated from the formula

$$M = R^2 \int_0^{2\pi} T_1 \cos\eta \, d\eta \tag{6}$$

The strain measures are related to the displacement components by the equations

$$\kappa_1 = -\frac{1}{R^2}\frac{\partial^2 w}{\partial \xi^2} \tag{7}$$

$$\kappa_2 = -\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \eta^2} + w\right) \tag{8}$$

$$\tau = -\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \xi \partial \eta} - \frac{3}{4}\frac{\partial v}{\partial \xi} + \frac{1}{4}\frac{\partial u}{\partial \eta}\right) = 0 \tag{9}$$

$$e_1 = \frac{1}{R}\frac{\partial u}{\partial \xi} \tag{10}$$

$$e_2 = \frac{1}{R}\left(\frac{\partial v}{\partial \eta} + w\right) = 0 \tag{11}$$

$$\gamma = \frac{1}{R}\left(\frac{\partial u}{\partial \eta} + \frac{\partial v}{\partial \xi}\right) = 0 \tag{12}$$

Here $R\xi$ denotes the distance measured along the axis of the undeformed tube, as shown in Figure 2, and (u, v, w) denote displacement components in the $(\xi, \eta, \text{radial})$ directions.

We suppose that the pressure loading $q(\eta)$ acting normal to the surface of the shell can be expressed as

$$q(\eta) = -K[w(\eta) + w(\eta + \pi)] - \frac{\delta R}{2}(1 + \cos 2\eta) \tag{13}$$

at $\eta = 0$ is the longitudinal curvature of the cylinder axis due to the beam-like bending. The terms in (15), (17), (19), (20) which are underlined are *secondary* quantities arising from ovalization of the cross section. Proper design requires that these quantities be made relatively small by having a large enough foundation modulus K.

3. APPLICATION TO DOUBLE HULL

As an example of the application of these formulas, let us consider the double hull sketched in Figure 1. An average hull section contains 3 plates each of thickness t and width b, as shown in Figure 5, so the average hull thickness is

$$h = 3t \tag{21}$$

The bending stiffnesses corresponding to this double hull geometry are

$$D_1 = \frac{7Eb^2t}{12(1-\nu^2)}, \quad D_2 = \frac{Eb^2t}{2(1-\nu^2)}. \tag{22}$$

Of interest for design purposes are the maximum stresses arising in the double hull. The magnitudes of the membrane stresses corresponding to the membrane forces (14) - (15) are less than

$$\sigma_{M1} = \frac{M}{3\pi t R^2} \tag{23}$$

$$\sigma_{M2} = \frac{\delta R^2}{4t} + \frac{\nu b^2 M}{12\pi(1-\nu^2)tR^4} + \frac{\delta R^2}{12t(1+\frac{2KR^4}{9D_2})} \tag{24}$$

The magnitudes of the bending stresses corresponding to the bending forces obtained from (4) and (16) - (17) are less than

$$\sigma_{B1} = \frac{bM}{7\pi(1-\nu^2)tR^3} + \frac{\nu\delta R^3}{7bt(1+\frac{2KR^4}{9D_2})} \tag{25}$$

$$\sigma_{B2} = \frac{\delta R^3}{6bt(1+\frac{2KR^4}{9D_2})} + \frac{\nu b M}{6\pi(1-\nu^2)tR^3} \tag{26}$$

From the architextural drawings of the double hull cross section sketched in Figure 1, we have computed the average values displayed in the Table of the various geometrical and material parameters in our formulas. The value of M is the maximum value of the

design hogging bending moment. I is the moment of inertia of the hull cross section about its centroid. If we choose the cylindrical cross section to have the same moment of inertia as the hull, this determines the radius of the cylinder to be $R = 25.75\,\text{ft}$. We suppose that the foundation stiffness K is the same as the bulkhead stiffness. From experiments on pyramidal truss cores typical of those which are proposed for the bulkhead structure, we have determined the average value of K shown in the Table.

Now let us put some of the numbers shown in the Table into the formulas (23) - (26):

$$\sigma_{M1} = 21.6 \tag{27}$$

$$\sigma_{M2} = 2.1 \quad + \quad \underline{\frac{.7}{1 + \frac{2KR^4}{9D_2}}} \tag{28}$$

$$\sigma_{B1} = 1.15 \quad + \quad \underline{\frac{3.2}{1 + \frac{2KR^4}{9D_2}}} \tag{29}$$

$$\sigma_{B2} = \underline{\frac{12.4}{1 + \frac{2KR^4}{9D_2}}} \quad + \quad .4 \tag{30}$$

Here all stresses are in units of ksi. Note that if there were no elastic foundation $(K = 0)$, the secondary stresses (those terms underlined in (28) -(30)) would be a sizeable fraction of the primary stresses. But $\frac{2KR^4}{9D_2} = 3733$, from the values in the table, so these secondary stresses are rendered negligible by the bulkhead stiffness.

4. CONCLUSION AND FUTURE WORK

Our analytical formulas indicate that the stiffness of the proposed bulkheads could be reduced by a factor of 10 without incurring significant secondary stresses in the double hull. A less bulky bulkhead design would have obvious cost and weight benefits.

This analysis models only the most fundamental aspects of the complex state of stress which could actually exist in the hull of a ship at sea. In future work the model should be refined to make its predictions correspond more closely to reality. Features which should be included are:

1. A more rectangular cross section corresponding to the actual hull shape. This could perhaps be included within the framework of the present analysis by conformal mapping techniques.
2. A more realistic applied loading, including torsion and internal pressures.
3. Discrete treatment of the bulkheads and other stiffeners. Also, allowance for nonuniform stiffness properties.
4. Geometric and material nonlinearities. This is essential for an ultimate strength analysis.

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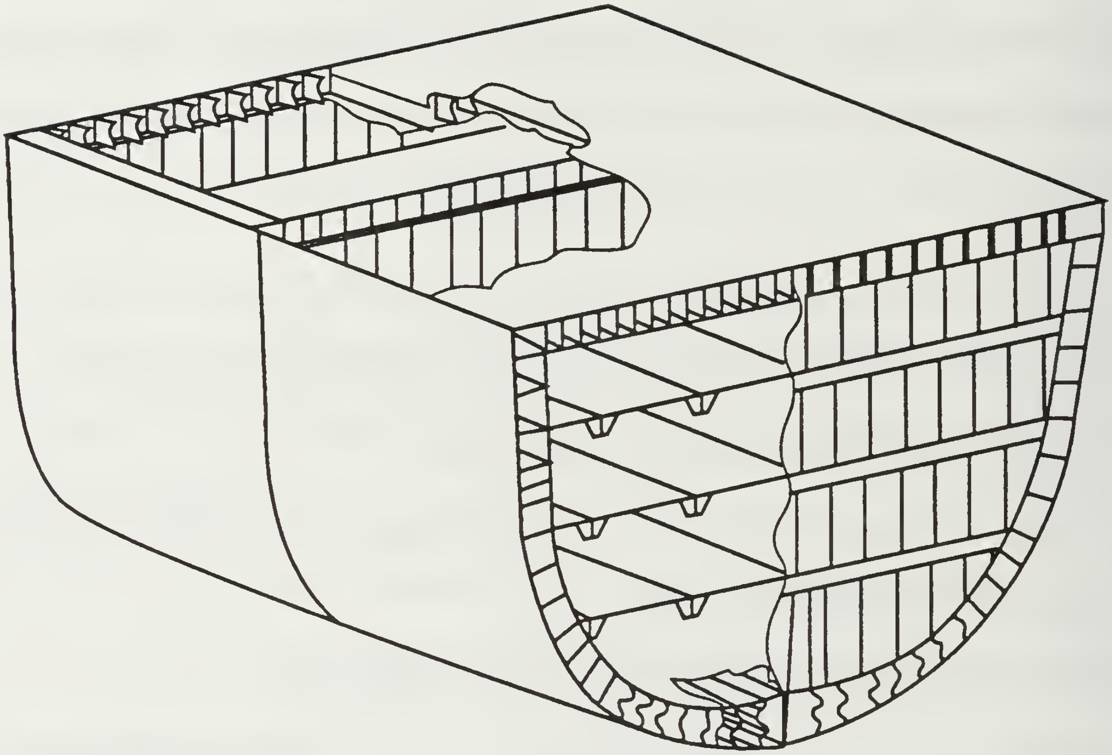


Figure 1: Double hull designed by David Taylor Research Center.

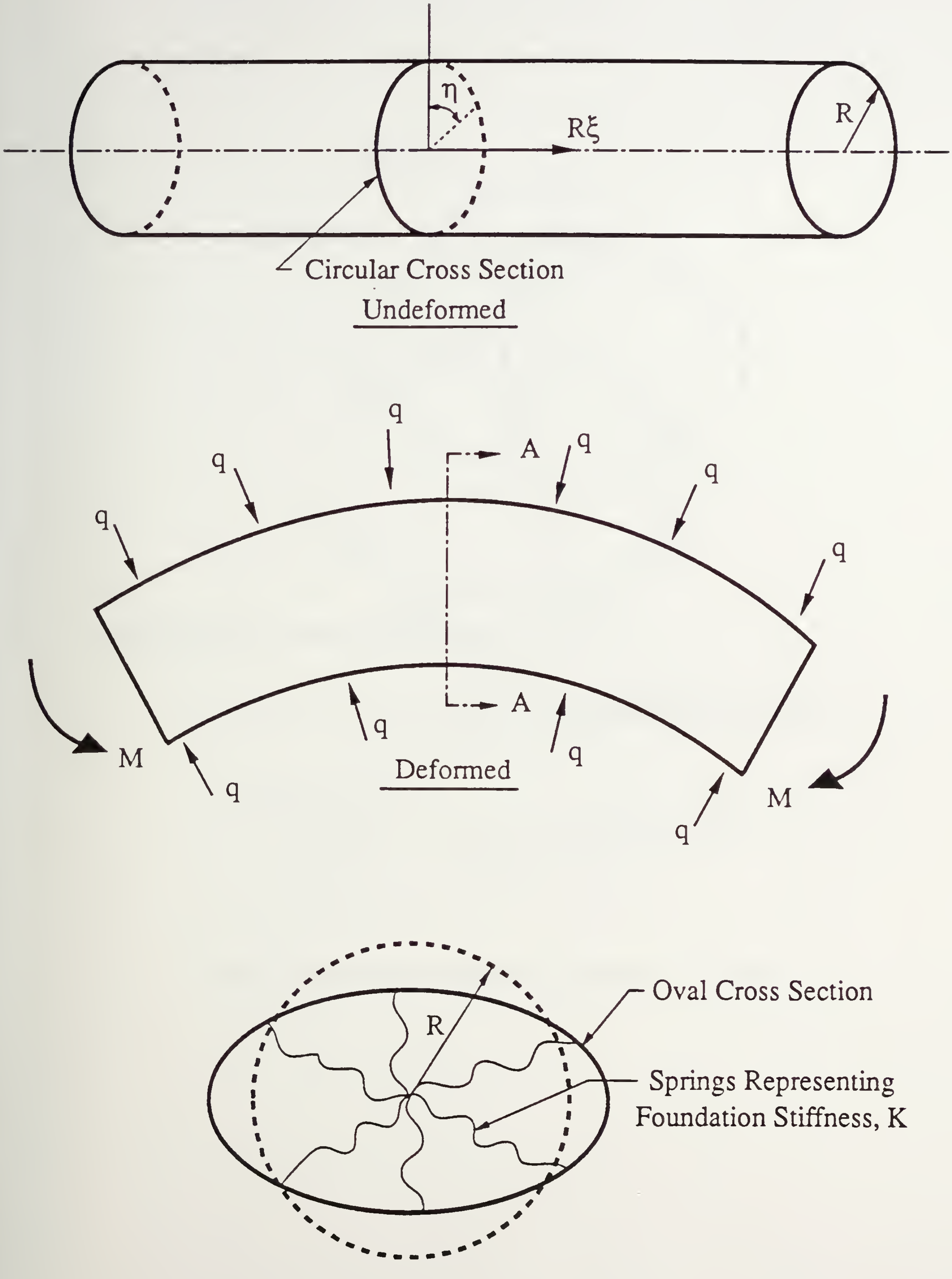
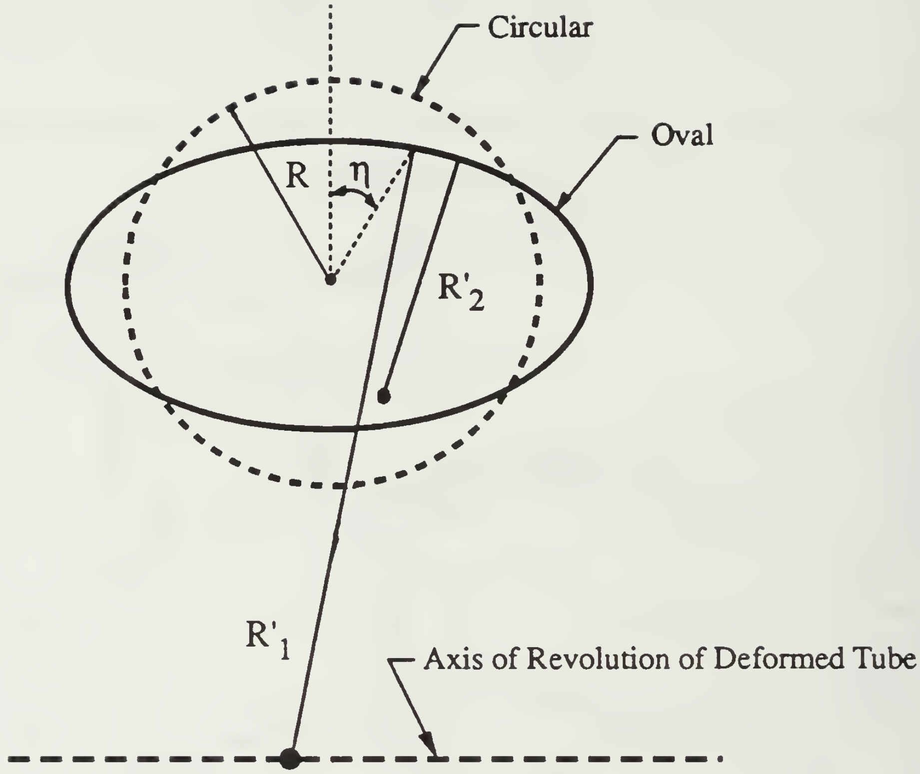
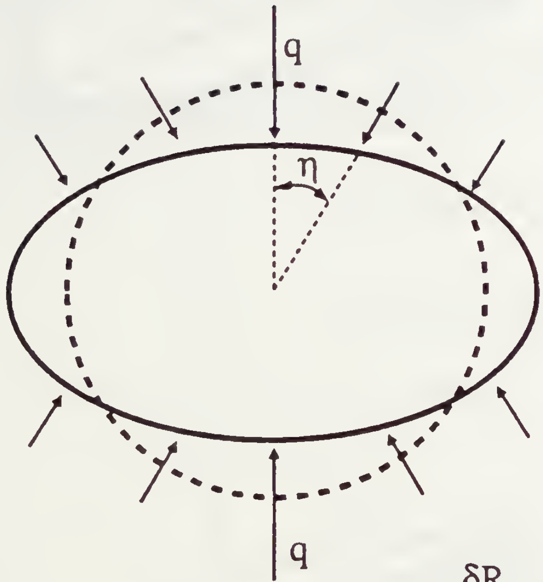


Figure 2: Idealized Model of Double Hull



$$\kappa_1 = \frac{1}{R'_1} \qquad \kappa_2 = \frac{1}{R'_2} - \frac{1}{R}$$

Figure 3: Geometrical Definitions of Curvature Changes



$$q = -\frac{\delta R}{2}(1 + \cos 2\eta)$$

Figure 4: Lateral Pressure Loading

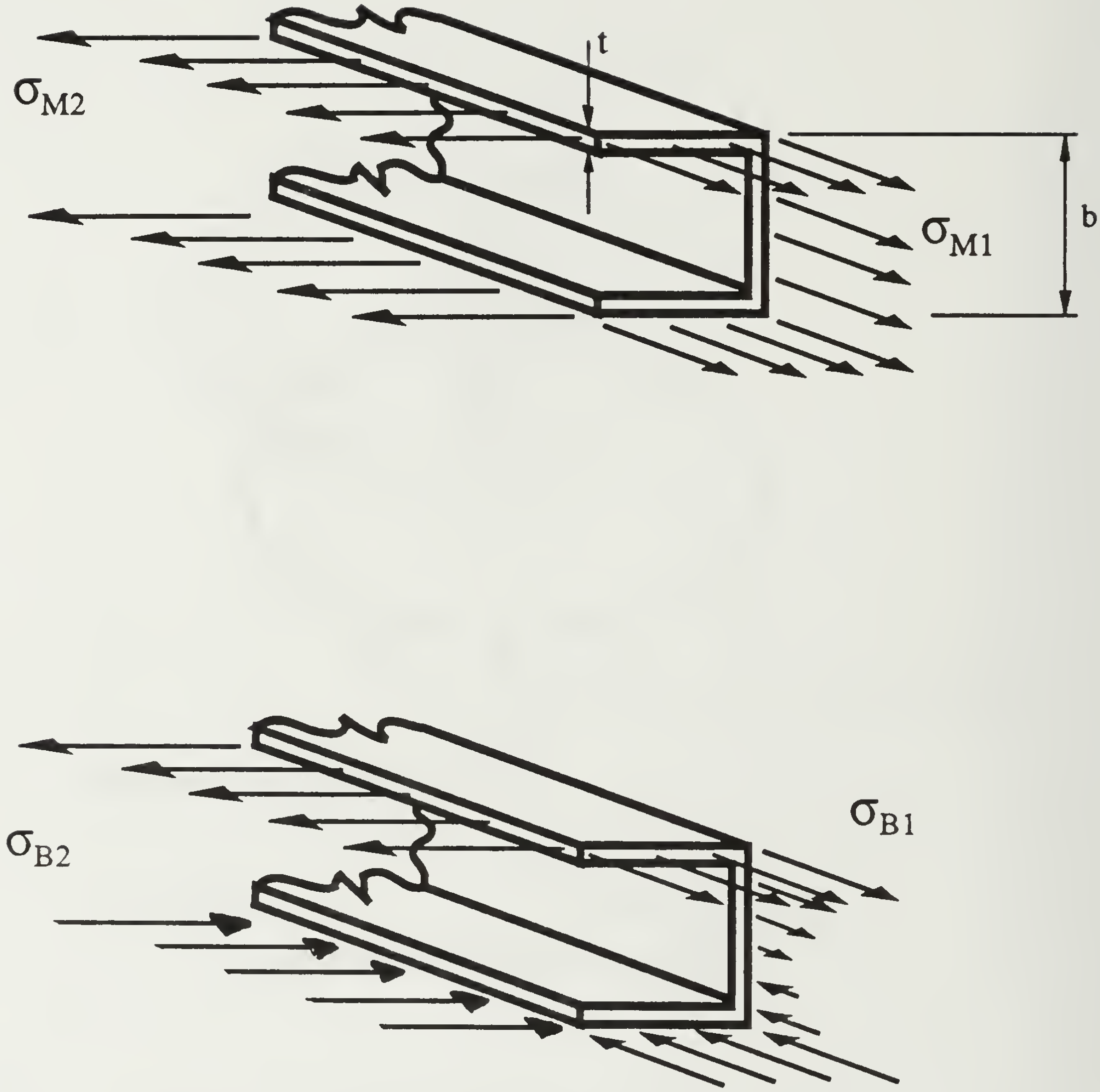


Figure 5: Stresses Acting in Double Hull Wall

Table 1: Typical Parameter Values

M	665,000 kip-ft
δ	0.0624 kip/ft^3
E	29,500 ksi
ν	0.3
t	0.41 in
b	35 in
$I=\pi R^3 h$	5500 ft^4
K	15 kip/in^3

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